I. 10. Stopping Rule for EM Algorithm Based on the Image Reconstructed by FBP Algorithm

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Introduction

Nowadays the Expectation Maximization (EM) algorithm is used instead of the Filtered Backprojection (FBP) algorithm in a lot of PET facilities. The reason of this shift is that EM algorithm provides better quality images compared to FBP algorithm. Images reconstructed by FBP algorithm have many streak artifacts and are poor to make clinical diagnosis. In contrast, EM algorithm does not exhibit such artifacts and provides clearer images. However, EM algorithm does not guarantee reliable quantification which is an important factor in PET imaging. Therefore, many PET facilities use images reconstructed by FBP algorithm when PET images are analyzed by compartment models. When analyzing by compartment models, the images require not only reliable quantification but also quality because good quality is essential to decide regions of interest (ROIs). The quantification problem for EM algorithm lies on the lack of a suitable stopping rule for the iteration procedure: Namely, it is indispensable to estimate how many iterations are required to guarantee good quantification. For such stopping rule, many methods are discussed and proposed\(^1,2\). In this paper, we propose a new approach of stopping rule by using the images reconstructed by FBP algorithm.

It is well-known that FBP algorithm shows reliable quantification properties for the chosen ROIs. If a stopping rule to guarantee the quantification is derived from images reconstructed by FBP algorithm, images reconstructed by EM algorithm are expected to have good quantification properties characterized by FBP algorithm and good quality originated from EM algorithm. Together, FBP and EM algorithm procedure take long time, but recent computer developments allow good performance to reconstruct images with both
algorithms in 2D mode. From these viewpoints, a stopping rule by using images reconstructed by FBP algorithm is expected to become a powerful method for reconstructing appropriate quantitative images by EM algorithm.

**Method**

Sinograms were acquired by a Monte Carlo simulation of a one ring PET scanner. Detector efficiency, scatter components and absorption were neglected in this simulation. Two kinds of phantom were used and their shape and radioactivity ratio are shown in Fig. 1. Each phantom is to evaluate the convergence tendency for radioactivity ratio (phantom 1) and hotspot size (phantom 2). The events were generated according with the radioactivity ratio by using random numbers and the total events assumed in this simulation were $10^6$ and $10^7$.

Images were reconstructed, from these sinograms, by FBP algorithm using Shepp-Logan filter and EM algorithm iterated till 50 times. For each image, after taking several ROIs covering all hotspot areas, the mean value and standard deviation of images were examined. A method to derive a stopping rule, from those values, was evaluated.

**Result and Discussion**

Fig. 2 (left) shows the ROI position of phantom 1. Fig. 3 shows the mean value and standard deviation of the images in each ROI at the case of total events $10^7$ and Fig. 4 shows at the case of total events $10^6$. Square points indicate the value obtained from the EM algorithm. Dashed and solid lines are the FBP and true values respectively.

When the number of iterations is increased, the mean value of images, reconstructed by EM algorithm, in each ROI converges to the one of true images regardless of the total events. At the case of total events $10^7$, the mean value of images reconstructed by EM algorithm for the ROI 3 is the fastest to converge and the one for the ROI 1 is the slowest. The mean value of images reconstructed by EM algorithm reached the one by FBP algorithm at 19 iterations in ROI 3 and 27 iterations in ROI 1. At the case of total events $10^6$, the convergence of mean value is same as at the case of total events $10^7$ and the mean value of images by EM algorithm reached the one by FBP algorithm at 20 iterations in ROI 3 and 30 iterations in ROI 1. Therefore, the necessary number of iterations for the mean value convergence becomes ROI 3<ROI 2<ROI 1.

At the case of total events $10^7$, though the standard deviation, in ROI 2 and 3, of images after a few of iteration is higher than the one by FBP algorithm, this result can be neglected because it appears at a very early stage of the image reconstruction process.
Thus, the standard deviation of images by EM algorithm till 50 iterations is less than the one by FBP algorithm. However, at the case of total events $10^6$, the standard deviation of images by EM algorithm in ROI 1 after 11 iterations is larger than the one by FBP algorithm and the ones in ROI 2 and 3 are considered as becoming larger than the one by FBP algorithm for more than 50 iterations because the shape of graph is upward-sloping curve. Thus, when total amount of events is small, it is understood that EM algorithm provides images reflecting the bad counting statistics and emphasizes the statistical noise according with the increase of iteration procedures. The case of few total events, in other words, when decreasing data acquisition time or radioactivity injected to patients, suggests the importance of a stopping rule.

Fig. 2 (right) shows the ROI position of phantom 2 and Fig. 5 shows the mean value and standard deviation of images in each ROI at the case of total events $10^6$ and Fig. 6 shows at the case of total events $10^7$. Details in figures are the same as above-mentioned Fig. 3 and 4.

When the number of iterations increases, the mean value of images by EM algorithm converges to the true one as the case of phantom 1. Necessary iteration number for the mean value of images by EM algorithm to converge to the one by FBP algorithm is ROI 2<ROI 1<=ROI 3 regardless of the difference of total events. For the standard deviation of images by EM algorithm, the importance of a stopping rule is shown for few total events because the shape of graph is upward-sloping curve same as the phantom 1.

A stopping rule should be derived from the dependence on the convergence tendency of the phantom shape and radioactivity ratio. However, only the fact that the hotspot with higher radioactivity ratio converges faster than the one with lower radioactivity ratio could be observed (Fig. 3 and 4) and the relation between convergence and phantom shape could not be observed (Fig. 6 and 7). At this point, a stopping rule results as follows: 1. Taking the ROI included all pixels in all hotspots and 2. Iteration procedure should be stopped when the mean value of images by EM algorithm reaches the one by FBP algorithm and then 3. If the standard deviation of images by EM algorithm is not a minimum, more iteration is necessary. However, attention must be paid not to increase the standard deviation in other ROIs after more iteration processes. Therefore, the comprehensive judgment for all ROIs or another solution is needed. For this problem, we are now considering a new algorithm such that some weighted factors are applied to the update pixels which have already converged.
Conclusion

In this paper, we performed Monte Carlo simulation and tried to derive a stopping rule for EM algorithm based on the images reconstructed by FBP algorithm. We proposed the stopping rule that iteration procedures should be stopped when the mean values of images by EM algorithm in the region of all hotspots arrive at the one by FBP algorithm. Fig. 7 shows, at the case of total events $10^6$, the true image of phantom 1, the images reconstructed by FBP algorithm and by EM algorithm according with the proposed stopping rule. The image reconstructed by EM algorithm has good quantification properties as the one by FBP algorithm and better quality than the one by FBP algorithm. However, when the image is analyzed more detailed, the standard deviation of images in B.G region has already become larger than the one at a few iterations before. For example, the standard deviation after 20 iterations (when the mean value of images by EM algorithm only at ROI 3 has converged to the FBP one) was about 14% and after global convergence was about 20%. This means that the image by EM algorithm at the proposed adequate iteration number was already degraded in the B.G region. For this point, we will make a new algorithm such as weighted EM algorithm. Moreover, the ROI selection and how many ROIs are needed in this stopping rule should be discussed more. We plan to develop automatically ROI taking system and judging the stopping rule.

References


Fig. 1. Phantom 1 (left) and 2 (right). Radioactivity ratios A:B:C:D are 1:3:4:5 (phantom 1) and 1:3:3:3 (phantom 2).
Fig. 2. The ROI positions of phantom 1 (left) and 2 (right).

Fig. 3. The mean value (left column) and standard deviation (right column) of phantom 1 for total events $10^7$. Top row is for ROI 1, middle for ROI 2 and bottom for ROI 3. Solid line is for true, dashed line is for FBP algorithm and square points are for EM algorithm.
Fig. 4. The mean value (left column) and standard deviation (right column) of phantom 1 for total events $10^6$. Top row is for ROI 1, middle for ROI 2 and bottom for ROI 3. Solid line is for true, dashed line is for FBP algorithm and square points are for EM algorithm.

Fig. 5. The mean value (left column) and standard deviation (right column) of phantom 2 for total events $10^7$. Top row is for ROI 1, middle for ROI 2 and bottom for ROI 3. Solid line is for true, dashed line is for FBP algorithm and square points are for EM algorithm.
Fig. 6. The mean value (left column) and standard deviation (right column) of phantom 2 for total events $10^6$. Top row is for ROI 1, middle for ROI 2 and bottom for ROI 3. Solid line is for true, dashed line is for FBP algorithm and square points are for EM algorithm.

Fig. 7. True image (left), image reconstructed with FBP algorithm (center) and image with EM algorithm (right) of phantom 1 for total events $10^6$. 