IV. 12. PET Imaging Based on the Fourier Rebinning Algorithm

Oishi Y., Ishii K., Yamazaki H., Matsuyama S., Watanuki S.*, Itoh M.*, and Orihara H.*

Department of Quantum Science and Energy Engineering, Tohoku University
Cyclotron and Radioisotope Center, Tohoku University*

Introduction

The quest for increased sensitivity in positron emission tomography (PET) has led to the interest of acquisition of all possible coincidence lines in a gantry. It has become feasible with the advent of commercial scanners equipped with retractable septa. This technique significantly improves the sensitivity over conventional slice-orientated methods (2-D PET), but the time for image reconstruction is much longer. This drawback must be solved in actual clinical application based on dynamic imaging or whole body imaging. The necessary time for 3-D image reconstruction can be significantly reduced with the use of the super computer. However, it cannot be always available from the side of cost. Therefore, we need some methods to speed up the 3-D reconstruction without using the super computer. Recently, a rebinning algorithm has been proposed as a promising approach for overcoming time-consuming images reconstruction in 3-D PET.

This report is especially focused on the Fourier rebinning (FORE) algorithm which was implemented in Shimadzu SET-2400W scanner at Cyclotron and Radioisotopes Center of Tohoku University (CYRIC). It is well known that images become very noisy in the edge region of axial direction because of lack of the rebinned data. We aimed at developing an algorithm which utilizes maximally images reconstructed with the FORE algorithm. A new developed algorithm should suppress noises in the edge slices and maintain the quantification of small spots.

Fourier rebinning algorithm

For a PET scanner with N rings, the data acquired in 3-D mode consist of $N^2$ sinograms, in which N direct sinograms ($\theta = 0$) and $N(N-1)$ oblique sinograms are included. The four variables $s$, $\phi$, $z$ and $\delta$ are defined as follows (See Fig.1): $s$ is the distance between the axis of the scanner and the projection line onto a transaxial plane, $\phi$ is an angle of the projection direction with respect to the Y-axis on the transaxial plane, $z$ is the axial coordinate of the middle point between two detectors. The $\delta$ coordinate is defined as the tangent of the angle $\theta$ between a line of response and the transaxial plane.
We take the Fourier transform of oblique and directed sinograms with respect to the parameters of $s$, $\phi$ and $z$. In the 3-D Fourier transform, $P(\omega, k, \omega_z, \delta)$, $\omega$ is the radial frequency, $k$ is the Fourier index, and $\omega_z$ is the axial frequency. The exact rebinning algorithm yields the following relation between the 3-D Fourier transforms of oblique and direct sinograms$^{3,4}$:

$$P(\omega, k, \omega_z, \delta) = \exp(-i\Delta \phi)P(\omega \chi, k, \omega_z, 0)$$

In this relation, $\Delta \phi$ is the phase shift. $\Delta \phi = k \arctan \alpha$, and $\chi$ is the frequency scaling:

$$\chi = \sqrt{1 + \alpha^2}$$

For the fast implementation, an approximation can be derived by considering the first-order truncation Taylor expansions in $\alpha$ of the phase shift and of the frequency scaling factor.

$$P(\omega, k, \omega_z, \delta) = \exp(-i\Delta \phi / \omega_p)P(\omega, k, \omega_z, 0)$$

As the only dependence on the axial frequency $\omega_z$ is a linear phase shift, it is possible to calculate the inverse 1-D Fourier transform of $P(\omega, k, \omega_z, \delta)$ (with respect to $\omega_z$),

$$P(\omega, k, z, \delta) = P(\omega, k, z - \frac{k \delta}{\omega_z}, 0)$$

This Fourier rebinning approximation relates the 2-D Fourier transform of an oblique sinogram $(z, \delta)$ to the 2-D Fourier transform of a slice shifted axially by a frequency-dependent offset $\Delta z = -k \delta / \omega$. The Fourier rebinning equation is considered as the frequency-distance relation stating that the value of $P$ at the frequency $(\omega, k)$ receives contributions mainly from sources located at a fixed distance $t = -k / \omega$ along the lines of integration. Note that $t$ represents the signed distance measured on the transaxial projection of line response (LOR) from the midpoint of the LOR. The Fourier rebinning does not require execution of an axial Fourier transform with respect to $z$. This property considerably simplifies the implementation not only because of the small number of Fourier transforms, but more importantly because the data need no longer be known for all values of $z$, and hence the truncated data do not need to be estimated$^2$.

After all projection data is rebinned to parallel slices on the frequency space, inverse 2-D Fourier transform is calculated to return sinogram $P_{2D}(s, \phi z, 0)$. Finally, 2-D image reconstruction is performed.

The new algorithm (the improvement of Median filter)

First, images reconstructed with the FORE algorithm were processed by a three-dimensional median filter in order to suppress noise. The median filter has two operation modes: New Median ver.1 and New Median ver.2. The first one uses a $3 \times 3 \times 3$ matrix of filter size (27 voxel). The second one is a three-dimensional cross joint filter (7 voxel).

Second, the new algorithms are set up with the criterion to subtract the filtered image
from the non-filtered image in order to avoid removing small hot spots. This criterion decides each pixel value in the final image. The criterion is as follow: if the pixel values, in the matrix of cross joint (7 voxel), of subtraction image are positive number, the pixel value in the center of cross joint is returned to the value before filtering in order to recover the removed hot spots. The pixel value keeps with the value processed by the filter when there is at least one negative pixel value in the matrix of cross joint. This concept is originated from the difference between hot spot and noise. When we image a point source in the detector ring, the corresponding hot spot will be evenly spread on the neighbor pixels due to spatial resolution of detectors used in a PET scanner. On the other hand, the noise profile almost always shows a signal alternating with high and low values irrespective of the spatial resolution of detector. So, if the pixel values of subtraction image are positive numbers in a region, the criterion judges that hot spot is removed in this pixel. On the other hand, if not, noise is removed.

Experimental

The 3-D PET data were obtained with the Shimadzu SET-2400W scanner, which has the axial view of 20cm and 32 slices, in CYRIC. Commonly, a lot of data collected with this PET system is sent to SX4/44R super computer in the computer center of Tohoku University. However, in this experiment, image reconstruction was performed using workstation (AlphaStation XP1000) in CYRIC for the purpose of estimating reconstruction time without super computer. Both the 3D-FBP and the FORE algorithms used in this experiment were implemented in the Shimadzu SET-2400W.

In the first experiment, mainly designed to evaluate image quality, the radioisotope injected in the pool-phantom (20cm of the diameter, 25cm of height) was F-18 and the total counts for scan time were about $1.7 \times 10^8$ counts. The new algorithms were applied for the image reconstructed with the FORE algorithm and 2D-FBP.

In the second experiment, the new algorithms were applied for clinical data of a cancer patient to evaluate the quantification checking hot spots smaller than filter size were removed. Data were acquired for 3 minutes after 1 hour from injection of FDG.

Result and Discussion

Processing time

Table 1 shows the time required for image reconstruction and filter processing. The time required for reconstruction in the FORE+2DFBP algorithm was found to be ten times less than that in 3D-FBP. In addition to this, the new algorithm does not lengthen the time of image reconstruction, because it is a simple algorithm.

Evaluation for image quality (for the pool-phantom experiment)

Figure 2 shows standard deviation for pixel value of the plane phantom in a 3- cm
circular region of interest (ROI). While the standard deviation of the FORE+2DFBP is higher than that of the 3D-FBP in all slices, that of the New Median ver.2 is almost same as that of 3D-FBP in the region of ± 23 slices (about 73% of all slices) from the center slice No.32 in the cylindrical scanner of SET-2400W. For the New Median ver.1, the standard deviation of almost all slices is lower than that of 3D-FBP. From these results, the quality of image is best improved when the New Median ver.1 was applied. This is caused by the size of Median filter. If Median filter of large size is applied, you can get an image of good quality, but if you will have a problem of removing small hot spots, as discussed below.

*Evaluation of Quantification (for the clinical data)*

Figures 3 and 4 are reconstructed images of clinical data. The former is at the slice No.30 and the latter is at the slice No.7. Comparing the images processed by New Median ver.1 and ver.2 with those processed by FORE+2DFBP, we can clearly see that many streak artifacts are removed by New Median filter processing. In addition to this, a comparison with the image processed by 3D-FBP let us to notice that hot spots are kept by New Median filtering. As shown in Fig. 4, the image of FORE+2DFBP is very noisy and cannot clearly reveal a hot spot. However, the hot spot can be distinguished from background if the new developed algorithms are applied.

Figures 5 and 6 show profile across small hot spots in the middle slice No.30 and the edge slice No.7 respectively. As shown in Fig. 5, while two hot spots are distinguished, a small hot spot of about 3 pixel (6mm) size are removed for a portion of 1 pixel. But such hot spot removal was not observed for the New Median ver.2. Fig.6 shows that both new algorithms suppress the noise and keep the value of relatively large hot spots even in the edge slice.

Removing hot spot is not a serious problem for this experiment, but it will be a big problem if better spatial resolution and quantification are required. So, we will study further noise reduction and quantification in filter processing of images reconstructed with FORE+2DFBP, and improve the algorithm of filter processing.

*References*

Table 1. Processing time for 63 slices

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Processing time</th>
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<tbody>
<tr>
<td>3D-FBP</td>
<td>4 hours</td>
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<tr>
<td>FORE+2DFBP</td>
<td>20 minutes</td>
</tr>
<tr>
<td>New Median ver.1 or ver.2</td>
<td>Less than 1 minute</td>
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Fig. 1. Variables used in FORE.

Fig. 2. Standard deviation in ROI of each slice.
Fig. 3. Images (at slice No.30) reconstructed with 3D-FBP (left) and FORE+2DFBP (mid-left) and processed by new Median ver.1 (mid-right) and New Median ver.2 (right).

Fig. 4. Images (at slice No.7) reconstructed with 3D-FBP (left) and FORE+2DFBP (mid-left) and processed by new Median ver.1 (mid-right) and New Median ver.2 (right).

Fig. 5. Hot spot profile at slice No.30.

Fig. 6. Hot spot profile at slice No.7.