I. 20. A Note on a Simplified Model of Energy Shift and Broadening in an Ion-Guide Isotope Separator

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1. Introduction and modeling

In an ion-guide isotope separator 1) the ions emerging from the exit hole of the target chamber together with a supersonic neutral helium gas flow experiences collisions with the gas during acceleration till the skimmer hole, thus being subject to energy shift and broadening 2).

In order to reduce these effects, especially, the energy broadening, a new idea of "squeezer ion guide" has been introduced 3). It seems, however, still useful to consider and analyze these effects analytically, although here we discuss only a simplest one-dimensional model of energy loss and broadening of a positive ion.

We assume a uniform density of the helium gas between the exit hole and the skimmer with a distance of \( D \) apart, and that the exit potential relative to the skimmer is \( V \), so that the kinetic energy of the single charged positive ion arriving at the skimmer is \( E_0 \) eV when there is no helium gas. When there is helium, however, the ion has a probability of collision with it and the ion energy is no more a definite quantity but becomes a stochastic one. Assume that the ion loses all its energy and stops when it collides with a helium gas atom; this assumption seems not unreasonable if we interpret the \( \lambda \) defined below as an effective quantity. We define the collision probability per unit distance by \( \lambda \) and the probability of the last collision falling in \([y, y+dy]\) by \( P_{\text{last}}(y)dy \). Since the probability of no collision for \([0, y]\) is \([1 - \lambda y/N]^N \) (for \( N \rightarrow \infty \)) = \( \exp(-\lambda y) \), we get

\[
E(y) = (y/D)E_0 \quad \text{and} \quad P_{\text{last}}(y)dy = \exp(-\lambda y) \lambda dy,
\]

where we measure \( y \) from the skimmer toward the exit hole, \( E(y) \) is the ion energy when it arrives at the skimmer after the last collision at \( y \); \( y \) is a stochastic quantity and so is \( E \) as stated above.

2. Probability function of \( E \)
We call $E$ the "energy affected by scattering" or simply "energy" and obtain its probability density function $P(E)$; we trace the scattering process in the inverse time direction. Here it is important to note that there is a finite probability of $\exp(-\lambda D)$ for $E = E_0$ as given by integrating eq.(1); this is just the probability of no collision at all. For $0 \leq y \leq D$ using the relation between $y$ and $E$, we get the probability density function of $E$ as

$$P(E)dE = \left[ k + 5(E - E_0) \right] \exp(-kE)dE \text{ with } k = \lambda D/ E_0. \tag{2}$$

Note the delta-function term in eq.(2); only with this term can we get the valid normalization, i.e. $\int P(E) dE = 1$ (for $[0, E_0]$).

3. The number of collisions $n$

On the other hand the number of collisions $n$ of an ion between the exit and skimmer and its statistics are given by the Poisson distribution;

$$P(n) = \left[ <n>^n/n! \right] \exp(-<n>), \tag{3}$$

where $<n> = \lambda D$ is the mean value of $n$, i.e. $<n> = \Sigma n P(n)$.

4. Mean and variance of $E$

For brevity we use $m$ in place of $<n>$. The mean value of $E$ can be calculated in the following as a function of $m$.

$$<E> = \int E \cdot P(E)dE \{ 1 - \exp(-m)/m \} E_0, \text{ with } m = <n> = \lambda D. \tag{4}$$

The variance of $E$, i.e. $\sigma^2(E)$ can be also obtained after some calculations:

$$\sigma^2(E) = <E^2> - <E>^2 = \left[ [1- 2m \exp(-m) - \exp(-2m)]/m^2 \right] E_0^2. \tag{5}$$

Now we discuss the behaviors of $<E>$ and $\sigma(E)$. We derive two limiting cases, i.e. $m \to \infty$ and $m \to 0$, which correspond to a high-pressure and a low-pressure limits, respectively. From eq.(4) it is easy to see that

$$E_0 = E_0 <n> \quad \text{; for } m \to \infty,$$

$$E_0 \quad \text{; for } m \to 0. \tag{6}$$

The result is reasonable and easy to understand intuitively.

For $\sigma(E)$ also we can derive from eq. (5);

$$E_0 = <E> \quad \text{; for } m \to \infty,$$

$$<E> \to \left\{ \right\} \tag{7}$$
\[ \sqrt{\frac{m}{3}} E_0 \quad ; \text{for } m \to 0. \]

This is also understandable qualitatively. The exact dependence of $\sigma(E)$ on $m$ [eq. (5)] is given in Fig. 1. The curve is consistent with the discussions above. It is interesting to note that $\sigma(E)$ is largest ($\sim 0.36 \, E_0$) at $m \sim 1$, i.e. when the mean collision number is around unity.

6. Conclusion

The present model is rather simple to be taken quantitatively, but we can still understand the qualitative kinematical features of the ion behavior between the exit hole and the skimmer hole. It is, however, worthwhile to elaborate the modeling in many ways, e.g. by using more realistic assumptions as well as resorting to experimental or numerical methods etc.

Eq. (6) and Fig. 1 suggest that the higher the helium pressure, the smaller the energy broadening, but we must be careful because the present model is rather simple and, moreover, a higher pressure results in a more difficult experimental condition.

We acknowledge Drs. T. Shinozuka and K. Morita for discussions.

References


Fig. 1. Dependence of energy broadening $\sigma(E)/E_0$ on the mean scattering number $m = \langle n \rangle = \lambda D$. 