I. 2. Elastic and Inelastic Nucleon Scattering on $^{28}\text{Si}$ in $E \leq 35$ MeV and Core-Polarization Effect Derived from a Dispersive Optical-Model Analysis

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The nucleon-nucleus optical potential is of importance not only for analyses of nuclear reaction for studying individual nuclear structure or reaction dynamics, but also for understanding of fundamental nuclear excitation, when the mean field for positive energies are extrapolated towards negative energies to obtain the shell-model potential.1-2) Furthermore, it is significant for investigating the charge-symmetry-breaking (CSB) component of the nucleon-nucleus scattering. Winfield and his collaborators have discussed 3) CSB based on proton and neutron scattering data on self conjugate N = Z nuclei, comparing the real volume integrals of the proton ($J_p$) and neutron ($J_n$). After corrected for Coulomb effects, averaged ($J_n - J_p$)/$A$ have some remaining evidences. However, their conclusions suffer from the ambiguities to estimate the core-polarization effects.

The mass operator $M(r,r';E + i\Delta)$ of one-particle moving in a nuclear mean field is expressed as 4):

$$M(r,r';E + i\Delta) = V_{H,F}(r,r') + N(r,r';E + i\Delta),$$  \hspace{1cm} (1)

where the first term is Hartree-Fock field, and the second core-polarization correction. From the view points of parametrization in optical potentials, the first and second teams yield, respectively, $V_{H,F}$ (volume real) and $\Delta V$ (real) plus $W$ (imaginary). As discussed in detail in Ref. 5, $\Delta V$ and $W$ are connected through the dispersion relation because of the analytical property of the optical potential. The imaginary potential may be separated into surface-peaked ($W_{SP}$) and Volume components ($W_V$). In Ref.5, it is pointed out that phenomenological optical model analyses with both surface-peaked and volume imaginary
potentials show that the former dominates at low energies, while the latter becomes important at energies of some tens MeV. Therefore, in the present neutron energy the volume component should be significant. As such, understanding of the imaginary volume term (W) in the optical potential is quite stringent for estimating the core polarization effects in nuclei. Further detailed studies of elastic and inelastic, dominant channel of which is the first excited 2+ state, nucleons scattering in a variety of their energy may give more accurate information for this volume imaginary term.

An extensive work for nucleons scattering on 28Si has been recently reported by Howell et al. 6) They carried out the experiment for elastic and inelastic scattering of polarized and unpolarized neutrons from 28Si in an incident neutron energy range between 8 and 17 MeV, and reanalyzed n + 28Si data over the 8-40 MeV energy range, including those for p + 28Si 7, but except for neutron inelastic data in En > 26 MeV. In this paper, we report elastic and inelastic neutron scattering experiments on 28Si using a monoenergetic 35-MeV neutron beam along with analyses by the optical model for neutron and proton scattering data over an incident energy range 15 through 35 MeV.

A neutron beam was obtained from the charge-exchange 7Li(p,n)7Be (Q = -1.644 MeV) at 0°, where natural lithium target was bombarded by a 37-MeV proton beams from the AVF cyclotron at Cyclotron and Radioisotope Center, Tohoku University. In order to measure the angular distribution of cross sections for scattered neutrons, the neutron beam axis was rotated in a vertical plane using a beam-swinging system. 8) Energy spread of the neutron beam was typically 650 keV, while the neutron flux used was 5 × 105/μC/sr at the position of the scattering target. Scattered neutrons were analyzed by time-of-flight (TOF) technique with a flight path of 30 m. Details of the TOF and neutron facilities have been published elsewhere. 9,10)

Figure 1 shows the angular distributions of scattered neutrons together with theoretical predictions calculated with coupled channel Born-approximation (CCBA) described in detail later on. The raw cross section are obtained by subtracting the contributions of the air and neutron source itself (data for this subtraction are taken in another run without target). Then the raw cross sections are corrected for the finite scatterer effect for neutron yields by the Monte Carlo simulation method; estimation for multiple scattering and attenuation, and angle averaging.

The optical potential is parametrization in terms of standard Woods-Saxon form as

\[ U_{opt} = -V_R f_R(r, R_v, a_v) - iW f_U(r, R_D, a_D) \]

\[ + 4\pi D_0 W_D \frac{d}{dr} f_w(r, R_D, a_D) + \left( \frac{h}{m\pi c} \right)^2 V_{so} \frac{1}{r} f_s(r, R_{so}, a_{so}) \], \hspace{1cm} (2)
where

\[ f_i (r) = \left( 1 + \exp \left( \frac{r - R_i}{a_i} \right) \right)^{-1}, \quad \text{and} \quad R_i = \left( 1 + \beta_2 Y_{20} (\Omega) \right) r_i A^{1/3}. \]

Parameters for the spin-orbit term are fixed to those obtained by Howell et al.\(^3\) through analyses for analyzing power data: \(V_{SO} = 6.0\) MeV, \(r_{SO} = 1.010\) fm, \(a_{SO} = 0.500\) fm, and \(\beta_{2,SO} = -0.53\). Coupled-channel analyses (CC) are carried out by the code ECIS79 by Raynal.\(^{11}\)

The present analysis is summarized as following:

1. By using presently obtained 35-MeV data, preliminary search for \(\beta_2\) without \(W_D\) has been carried out. The results are \(\beta_2 = +0.2572\) with \(\chi^2/N = 1.3\), and \(\beta_2 = -0.4583\) with \(\chi^2/N = 0.8\). The present \(\chi^2\) and previously reports \(^4\) strongly support for the sine of \(\beta_2\) to be negative.

2. Secondary, with the same data, search for the geometrical parameters are carried out. Under the condition of \(r_V = r_W\), we obtain \(r_V = 1.21\) fm, \(a_V = 0.6185\) fm, and \(a_W = 0.6268\) fm. These resultant diffuseness parameters are quite close to each other. This may support the validity of the assumption made for \(r_V\) (or \(W\)), and the numerical result for \(r_V\).

3. Based on these geometrical parameters for the volume imaginary term, we search for the diffuseness parameters of the surface imaginary term at lower energy, where the volume term plays minor role as described before. With the data at \(E_n = 16.9\) MeV in Ref. \(^4\), we obtain \(a_D = 0.43\) fm.

4. Since \(r_D\) has been reported to be energy dependent \(^2\), parameter search for \(r_D\) are made by neutron scattering data at \(E_n = 14.8, 16.9, 20.0, 21.7, 26.0\) and 35.0 MeV, the result by which is;

\[ r_D = 1.49 - 0.012E_n. \]

5. Similarly, \(r_V\) may be slightly energy dependent. We obtain;

\[ r_V = 1.1863 + 0.0017E_n. \]

6. Finally, with these geometrical parameters discussed above, search for the strength parameters of \(V_R, W_V, W_D\) and \(\beta_2\) are carried out. The results are listed in TABLE 1, along with the 6 incident neutron energies indicated before while the optical potential parameters thus obtained for the 35-MeV neutron scattering on \(^{28}\)Si are tabulated in TABLE 2, and simultaneous fitting to elastic and inelastic differential cross sections are illustrated in Fig. 1.
Based on the numerical results listed in these Tables, we discuss the mean field for the neutron and $^{28}$Si system in terms of the volume integral of each potentials. We can define the familiar volume integral of a deformed potential $U(\mathbf{r}, \Omega)$ as following,

$$J/\mathbb{A} = \frac{1}{\mathbb{A}} \int U(\mathbf{r}, \Omega) \, d\mathbf{r},$$

(3)

by which we are able to discuss less ambiguously independent of the fine structure of uncertainties for geometrical parameters. Furthermore, this radial moment satisfy the following dispersion relation:

$$J_V = J_{HF}(E) + \frac{P}{\pi} \int_{-\infty}^{+\infty} \frac{J_W(E')}{E' - E} \, dE',$$

(4)

where $J_{HF}(E)$ correspond to the Hartree-Fock component of the mean field, and $P$ denotes the principle value of integral, while $J_W(E)$ is composed of integrals for imaginary-volume and imaginary-surface components in the optical potential.

In order to perform the integration in Eq. (3), and to obtain the real part due to core polarization explicitly, we parametrize the energy dependence of the potential as a form suggested by Brown and Rho $^{12}$) and applied successfully by Mahaux and Sartor $^{1)}$;

$$-J = -C_1 \frac{(E_n - E_F)^2}{(E_n - E_F)^2 + C_2^2},$$

(5)

where $E_F$ is the Fermi energy (-12.8 MeV for the present case). The parameters $C_1$ and $C_2$ is those should be searched empirically. By fitting the experimental results for six neutron energies, we obtain the magnitudes of $C_1$ and $C_2$ to be; for the total imaginary part, $C_1 = 134$ MeV-fm$^3$, and $C_2 = 20$ MeV, while for the volume imaginary part $C_1 = 134$ MeV-fm$^3$, and $C_2 = 55$ MeV. Starting from the imaginary potential, which have been obtained from inelastic cross sections, we are able to evaluate through the dispersion relation the real part due to particle-hole excitation or core-polarization. Energy dependence for these volume integrals, thus obtained, is illustrated in Fig. 2. It should be remarked that the real surface component $J(\Delta V_S)$ plays indeed minor role in the neutron energy region presently investigated (shadow region of Fig.2).

Now we are able to evaluate the real part/J[\Delta V(E)] due to core-polarization separately from that due to the Hatree-Fock component $J[V_{HF}(E)]$ as shown in Fig. 3. Curves in the
figure are exponential fittings. Energy dependence of the volume integral for the real potential is parametrized as following:

\[ J_V(E) = -462.1 \exp \left\{ -5.33 \times 10^{-3}(E_n - E_F) \right\} - 134 \times 55 \frac{(E_n - E_F)}{(E_n - E_F)^2 + 55^2} \text{ [MeV-fm}^3] \] 

(6)

Alternatively, those for imaginary volume and surface components are expressed similarly. As a consequence, based on the dispersion relation energy dependence for the depth parameters in the optical potential has been obtained in a wide range of neutron energy. Especially, for the real part we have separated the Hatree-Fock component from those due to other higher-order in the nuclear mean field, the dominant part for which is suspected to

Since the $^{28}\text{Si}$ target is one of the $N = Z$ self-conjugate nuclei, the isovector terms in a potential vanish. Thus, it is plausible to compare neutron-nucleus scattering with proton-nucleus one, taking into accounts some differences; Fermi energy for proton is -7.2 MeV instead of -12.8 MeV for neutron, while Coulomb energy difference is 5.6 MeV. Proton-$^{28}\text{Si}$ elastic- and inelastic-scattering data by DeLeo and his collaborators have been analyzed in a incident proton energy region 18.7 MeV through 40.2 MeV in a similar manner for neutrons as described previously; except Coulomb potential, where we adopted $r_C = 1.314 \text{ fm}$. As for the geometrical parameters other than Coulomb potential, we have adopted the same values with those for neutrons in order to find potential depth parameters by fitting the experimental data. Thus, we have found equivalent information for the proton $^{28}\text{Si}$-potential to that for the neutron-$^{28}\text{Si}$ potential based on the dispersion relation.

As pointed out earlier, the difference of volume integrals between neutron-nucleus and proton-nucleus may give a measure of charge symmetry breaking in the nuclear mean field. The Coulomb corrected $(J_n - J_p)/A$ values obtained for $^{28}\text{Si}$ in the present analyses indeed exhibit some remaining effects as shown in Fig. 5. However, the line in Fig. 5, showing the difference of the core-polarization effect between n-$^{28}\text{Si}$ and p-$^{28}\text{Si}$, explain quite reasonably, at least in higher incident energy region including presently investigated

In conclusion, we have measured neutron inelastic scattering at the highest energy of 35 MeV, and have made a dispersive optical model analyses for this data together with neutron data in $14.8 \leq E_n \leq 26.0 \text{ MeV}$, and those of protons in $18.7 \leq E_p \leq 40.2 \text{ MeV}$. The imaginary part of the optical potential obtained through inelastic data has enabled us to evaluate the real part due to core-polarization by the aids of the dispersion relation. The $(J_n - J_p)/A$ for $^{28}\text{Si}$ may due to core-polarization rather than charge symmetry breaking.

References

11) Raynal J., the code ECIS79 (unpublished).

Table 1. Optical potential strength parameters obtained in the present analysis.

<table>
<thead>
<tr>
<th>E_n (MeV)</th>
<th>V_V (MeV)</th>
<th>W_V (MeV)</th>
<th>W_D (MeV)</th>
<th>( \beta_2 )</th>
<th>( \chi^2/N )</th>
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</thead>
<tbody>
<tr>
<td>14.8</td>
<td>45.98</td>
<td>0.08</td>
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<td>16.9</td>
<td>45.46</td>
<td>-2.74</td>
<td>9.6505</td>
<td>-0.38</td>
<td>50.7</td>
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<tr>
<td>20.0</td>
<td>44.78</td>
<td>3.54</td>
<td>4.6876</td>
<td>-0.43</td>
<td>70.7</td>
</tr>
<tr>
<td>21.7</td>
<td>44.18</td>
<td>-0.09</td>
<td>8.4170</td>
<td>-0.39</td>
<td>42.0</td>
</tr>
<tr>
<td>26.0</td>
<td>40.96</td>
<td>2.40</td>
<td>8.7412</td>
<td>-0.51</td>
<td>11.2</td>
</tr>
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<td>35.0</td>
<td>39.68</td>
<td>9.05</td>
<td>2.3783</td>
<td>-0.52</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Table 2. Best fit optical potential parameters for 
\( n + ^{28}\text{Si} \) system at \( E_n = 35 \text{ MeV} \).

<table>
<thead>
<tr>
<th>V_V (MeV)</th>
<th>r_V (fm)</th>
<th>a_V (fm)</th>
<th>( \beta_{2V} )</th>
</tr>
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<tbody>
<tr>
<td>40.3</td>
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<td>-0.5407</td>
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<table>
<thead>
<tr>
<th>W_W (MeV)</th>
<th>r_W (fm)</th>
<th>a_W (fm)</th>
<th>( \beta_{2W} )</th>
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</thead>
<tbody>
<tr>
<td>9.2179</td>
<td>1.21</td>
<td>0.6281</td>
<td>( \beta_{2V} = \beta_{2W} )</td>
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</table>

<table>
<thead>
<tr>
<th>W_D (MeV)</th>
<th>r_D (fm)</th>
<th>a_D (fm)</th>
<th>( \beta_{2D} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.9943</td>
<td>1.08</td>
<td>0.43</td>
<td>0.1251</td>
</tr>
</tbody>
</table>
Fig. 1. Center of mass cross sections for elastic and inelastic (leading to the first-excited 1.7789 MeV, 2+ state) neutron scattering from $^{28}$Si. The curves are CC fits to the data as discussed in the text.
Fig. 2. The upper diagram represents the energy dependence of the volume integrals per nucleon calculated by Eq. (4) for volume and surface-peaked components together with their sum for the imaginary part of the mean field. The lower curves are those for the real part (other than Hartree-Fock component) of the mean field derived, through the dispersion relation, from the curves of upper diagram.
Fig. 3. Open (Ref. 6) and closed (present) circles denote 'experimental' volume integrals of the real component. The triangle points denote those obtained by subtracting the corresponding $J[\Delta V(E)]$ in Fig. 3; thus they are $J[V_{HF}(E)]$ fitted by an exponential dash and dotted curve. Solid curve is sum of $[V_{HF}(E)]$ and $J[\Delta V(E)]$ in Fig. 3.

Fig. 4. Differences of the real volume integrals per nucleon for neutron and proton scattering from $^{28}\text{Si}$. The curves in the figure represents a theoretical interpretation for these differences in terms of core-polarization derived from $J[\Delta V]$ in Fig. 3.