I. 8 A Shell-Model Study of Gamow-Teller Matrix Elements in the 0f\(_{7/2}\)-Shell Nuclei

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A systematic analysis of the Gamow-Teller (GT) matrix elements in the sd-shell region has been carried out by Brown, Chung and Wildenthal.\(^1\)\(^-\)\(^2\)

In the fp-shell region such a systematic analysis has not been done yet. Recently, in this region experimental values with sufficient precision have been accumulated. Thus a systematic study of GT matrix elements in the 0f\(_{7/2}\)-shell region is now highly required.\(^3\)

In our shell-model calculation the configuration space was taken as

\[
\ell^n + \ell^{n-1}r + \ell^{n-2}r^2,
\]

where \(\ell\) denotes the 0f\(_{7/2}\) orbit and \(r\) denotes one of the 1p\(_{1/2}\), 0f\(_{5/2}\) and 1p\(_{3/2}\) orbits. The number of active particles is \(n=A-40\) assuming an inert core of 40Ca. As the two-body effective interaction the Kuo-Brown\(^4\) matrix elements were basically adopted. We introduced a weak mass-number dependence of the single-particle energy.

Before we compare the theoretical values with the experimental ones, we have examined the validity of the configurational assumption in terms of \(N_{\text{jump}}\) which is the maximum number of particles excited from the 0f\(_{7/2}\) orbit to the upper fp-shell. For this purpose the theoretical values of the GT matrix elements, \(B(GT)^{1/2}\), are calculated with various \(N_{\text{jump}}\) models as shown in Fig. 1. The \(B(GT)^{1/2}\) values are calculated with the free-nucleon single-particle GT matrix elements (hereafter denoted as "free" SPGT),

\[
<j||\text{Op}(GT)||j',\text{free}> = \left|g_A/g_V\right|<j||\text{Op}_s||j'>,
\]

where \(\left|g_A/g_V\right| = 1.251(9)\).\(^5\) It is noted that the full fp-shell calculations are feasible by our shell-model code\(^6\) for \(A=46\) nuclei.

It is clearly seen in Fig. 1 that the values of \(B(GT)^{1/2}\) for increasing \(N_{\text{jump}}\) are converging to the assumed "full-space prediction" smoothly, and that the \(N_{\text{jump}} = 2\) model gives values closer to the full-space predictions than the \(N_{\text{jump}} = 1\) model does.

The experimental GT matrix elements are derived in the same manner as in Ref. 2 and as tabulated in Ref. 3. In total fifty-two GT transitions observed in the 0f\(_{7/2}\)-shell nuclei are investigated using the present model. In Fig. 2 we compared the experimental \(B(GT)^{1/2}\) values with the theoretical ones for the
ground-to-ground mirror transitions. Strong reductions of experimental \( B(GT)^{1/2} \) in comparison with those of the \( N_{\text{jump}} = 0 \) model are seen, especially, for the decay from nuclei near \( A=55 \) like \( ^{56}\text{Ni} \). By taking into account the particle excitation from the \( 0f_{7/2} \) orbit, i.e. with the \( N_{\text{jump}} = 1 \) and 2 models, these reductions can be reproduced qualitatively.

In order to see the reduction mechanism of \( B(GT)^{1/2} \) by particle excitation we decompose \( B(GT)^{1/2}_{\text{th}} \) into its components, \( m(j,j') \), which is a product of the one-body transition density\(^3\) and the SPGT. We plot the values of \( m(0f_{7/2},0f_{7/2}), m(0f_{7/2},0f_{5/2}) \) and \( m(1p_{3/2},1p_{3/2}) \) for the ground-to-ground mirror transitions in Fig. 3. Other values of \( m(j,j') \) are negligibly small and are not shown in Fig. 3. The large deviation of \( m(0f_{7/2},0f_{7/2}) \) from the single-particle value of 1.42 suggests a strong seniority mixing in the \( J^\pi = 7/2^- \) wave functions even with the \( N_{\text{jump}} = 0 \) model. Here it is noted that at the beginning of the \( 0f_{7/2} \) shell the seniority mixing results in a reduction of \( B(GT)^{1/2} \) even stronger than the reduction due to the spin excitation effect which comes from the \( m(0f_{7/2},0f_{5/2}) \) component. However, the latter effect dominates as the \( 0f_{7/2} \) shell is being filled.

As Fig. 2 shows, the theoretical "free" values of \( B(GT)^{1/2} \) are generally larger than the observed ones. This tendency provides an important subject of the quenching of the GT matrix element. A simple improvement for getting a better agreement between the shell-model prediction and the experimental values is obtained by introducing a state-independent quenching factor of all the SPGT or, equivalently, a renormalization\(^5\) of the axial-vector coupling constant \( g_A \). In the following we derived an average state-independent renormalization factor defined as

\[
\rho = (1/N) \sum_i \rho_i \quad \text{with} \quad \rho_i = \frac{B(GT;i)}{B(GT;i)_{\text{th}}^{1/2}} \quad \text{free,}
\]

where \( N \) is the total number of transitions adopted and \( B(GT;i)_{\text{th}}^{1/2} \) free denotes the theoretical GT matrix element based on the free SPGT. For this purpose twenty-five transitions are selected from \( A = 43-55 \) according to the criterion of transition strength employed by Wilkinson.\(^5\)

The results are shown in Table 1 together with the estimated standard deviation, \( \sigma(\rho) \). The renormalization factor is \( \rho = 0.748(27) \) for the \( N_{\text{jump}} = 2 \) model. It is noted that the value of \( \rho \) is almost constant through the \( sd- \) and \( fp- \) shell regions, since the corresponding value for \( sd- \) shell calculations is \( \rho_{sd} = 0.76(3) \).\(^2\) The similarity of these two values of renormalization factor suggests mass-number independence of the quenching effect through the \( sd- \) and \( fp- \) shell nuclei. In this respect determination of the renormalization factor \( \rho \) for heavier nuclei using reliable shell-model wave functions is quite an interesting subject.
References


Table 1. Renormalization factor $\rho$ and estimated standard deviation $\sigma(\rho)$ obtained for different models$^a$.

<table>
<thead>
<tr>
<th>$N_{\text{jump}}$</th>
<th>$\rho$</th>
<th>$\sigma(\rho)$</th>
<th>$\sigma(\rho=1)^a$</th>
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<tr>
<td>0</td>
<td>0.590</td>
<td>0.036</td>
<td>0.089</td>
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<tr>
<td>1</td>
<td>0.766</td>
<td>0.037</td>
<td>0.059</td>
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<td>2</td>
<td>0.748</td>
<td>0.027</td>
<td>0.057</td>
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<tr>
<td>$2+\text{full}^b$</td>
<td>0.752</td>
<td>0.026</td>
<td>0.056</td>
</tr>
</tbody>
</table>

$^a$ Also the root-mean-square deviation of theory from experiment obtained when the renormalization is not introduced is given (denoted as $\sigma(\rho=1)$); see the text.

$^b$ The full fp-space wave functions are used for nuclei with $A \leq 46$ instead of the $N_{\text{jump}}=2$ wave functions.
Fig. 1. Model dependence of the theoretical GT matrix elements, $B(GT)^{1/2}_{\text{free}}$, based on the free-nucleon single-particle matrix elements. The models are characterized by the maximum number, $N_{\text{jump}}$, of particles excited from the $0f_{7/2}$ orbit to the upper fp-shell. The $N_{\text{jump}} = 0$ model is the same as the $(0f_{7/2})^N$ model, and the $N_{\text{jump}} = 4, 5,$ and 6 models are equivalent to the full fp-space models for $A=44, 45$ and 46, respectively. Note that the calculated values nearly lie on the horizontal axis for $N_{\text{jump}} = 1-5$ in the $^{45}\text{Ca}(7/2^-) + ^{45}\text{Sc}(7/2^-)$ transition.

Fig. 2. The theoretical GT matrix elements, $B(GT)^{1/2}_{\text{free}}$, in comparison with experiment for the ground-to-ground mirror transitions in the $0f_{7/2}$ nuclei. The maximum number of excited particles, $N_{\text{jump}}$, is indicated.
Fig. 3. The single-particle components, m(j,j'), for the ground-to-ground mirror transitions. The theoretical GT matrix elements B(GT)$_{1/2}$ free, are composed of m(j,j'). Only the major components are shown.