I. 9 Quasifree Electron Bremsstrahlung Induced by 20-MeV Proton Impact


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Continuum x rays produced by bombardments of gaseous or solid targets with heavy-charged particles or heavy ions have been interpreted in terms of molecular-orbital (MO) x rays, radiative electron capture (REC), radiative ionization (RI), and secondary-electron bremsstrahlung (SEB). In a case of heavy-ion impact, direct processes such as MO and REC play an important role, while multi-step processes such as RI and SEB play a predominant role in light-ion impact. Recently, we have observed a new continuum x-ray component coming from a special RI process in bombardments of Be, C, and Al targets with 6-40 MeV protons. The high-energy-end point of this component changes depending on the proton energy and is in agreement with $\frac{1}{2} m_e V_p^2$, where $m_e$ is the electron mass and $V_p$ is the projectile velocity. From this behavior, these continuum x rays have been well explained in terms of bremsstrahlung produced by orbital electrons scattered in the projectile-Coulomb field. We have obtained satisfactory agreement of the experimental results with calculated bremsstrahlung produced in the projectile frame assuming that the orbital electrons are free and at rest; we called these continuum x rays the quasifree electron bremsstrahlung (QFEB). Since the QFEB is a process occurring in the projectile frame, the Doppler effect is expected to appear in the spectrum; in the previous paper, this effect has been observed in the projectile-energy dependence of the spectrum. In this report, angular distributions of the QFEB from a Be target bombarded with 20-MeV protons are measured and the results is compared with theoretical calculations which take into account the correction for the Coulomb deflection. The effect of velocity distribution of the orbital electron on the spectral shape near the high-energy-end point of QFEB is calculated and is discussed in connection with the experimental result.

Beams of 20-MeV protons from Tohoku-University cyclotron bombarded a self-supporting Be target of 46-mg/cm² thickness, which was set in a scattering chamber newly built for measuring angular distributions and installed in the beam line described in Ref. 11. This chamber of 40-cm inner diameter has a

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sliding membrane with a window for the x-ray detector and is designed so as to
the angle between the target surface and the direction of x-ray detection is
always kept constant. Hence, the self-absorption, absorptions of x rays in the
air-path and windows are same at all angle of detection and the error to take
account of absorption corrections cancels out each other. Continuum x rays from
the Be target have been measured with an ORTEC Si(Li) detector at the directions
of $\theta_L = 50^\circ, 70^\circ, 90^\circ, 100^\circ, 120^\circ, 130^\circ, 140^\circ$ and $148^\circ$. In order to avoid pile-
up effect, counting rates have been kept at about 100 cps.

Production cross sections for continuum x rays, measured at angles $\theta_L = 50^\circ-
148^\circ$ and corrected for the detection efficiency and absorptions, are shown in
Fig. 1, where the ordinate represents the production cross section multiplied
by the x-ray energy — $\sigma \hbar \omega$; this is a scaling based on the x-ray-energy depen-
dence of bremsstrahlung — $\sigma \propto \frac{1}{\hbar \omega} f(\hbar \omega)$. Experimental errors have been esti-
mated to be 15 \% at about 6 keV and 20 \% at about 15 keV from the following un-
certainties: target thickness, 10 \%; detector efficiency and absorption correc-
tions, 11 \%, and counting statistics, 0.2 \% and 14 \%, respectively, at 6 keV and
15 keV.

The production cross section for electron bremsstrahlung, including the
first-order relativistic effect, is generally given by$^{12}$

$$
\sigma^{QEB}_{CM}(T_x, \hbar \omega, \theta) = \frac{z_p^2}{\pi} \frac{z_T^2}{\alpha \frac{m_e^2}{m_p}} \frac{C^2}{T_x} \sigma \propto \frac{1}{\hbar \omega} f(\hbar \omega) \\
\left[ \sin^2 \theta + \frac{1}{4}(1+T)(3\cos^2 \theta-1) \right] \log \frac{1+\sqrt{T}}{1-\sqrt{T}} - \frac{1}{2} \sqrt{T}(3\cos^2 \theta-1) \\
+ \frac{\hbar \omega}{2} \cot \theta \left[ (7-T)\sin^2 \theta + \frac{1}{2}(\cos^2 \theta-\frac{3}{2}\sin^2 \theta)(10T+3-3T^2) \right] \times \\
\log \frac{1+\sqrt{T}}{1+\sqrt{T}} - 2\sqrt{T}\cos\theta(\cos^2 \theta-2\sin^2 \theta), \text{ for } \hbar \omega \leq T_x \\
= 0, \text{ for } \hbar \omega > T_x. \quad (1)
$$

Here, $z_p$ and $z_T$ are the atomic number of the projectile and the target nucleus,
respectively, $\alpha$ is the fine-structure constant, $m_e C^2$ is the rest energy of electron,
$\hbar \omega$ is the x-ray energy, $\theta$ is the emission angle of x rays with respect to
the incident electron, and $T_x$ is the kinetic energy of orbital electron relative
to the projectile. Considering that the projectile mass is large enough in
comparison with the electron mass, we approximately obtain

$$
T_x = \frac{1}{2} m_e v_p^2, \quad (2)
$$

and $T$ and $\beta$ are defined by

$$
\beta = \frac{2T_\perp}{\sqrt{m_e C^2}}, \quad T = \frac{T_x - \hbar \omega}{T_x}, \quad (3)
$$
Further, \( g(\xi, \xi_0) \) is the correction term for the Coulomb deflection and is given by Sommerfeld\(^{12} \) by

\[
g(\xi_0, \xi) = \frac{\xi}{\xi_0} \frac{1 - e^{-2\pi \xi_0}}{1 - e^{-2\pi \xi}},
\]

with

\[
\xi_0 = z \frac{R_y}{T} \frac{V_y}{T}, \quad \xi = z \frac{R_y}{T - \hbar \omega},
\]

(4)

where \( R_y \) is the Rydberg constant.

The cross section \( \sigma_{QFEB}^{CM} \) has a finite value at the high-energy-end point \( \hbar \omega = T_r \) because of the correction factor \( g(\xi, \xi_0) \), whereas the PWBA calculation of \( g(\xi_0, \xi) = 1 \) gives \( \sigma_{QFEB}^{CM} = 0 \) at \( \hbar \omega = T_r \). It will be shown below that this difference between the two calculations plays an important role in the behavior of QFEB spectrum near the end-point energy.

Equation (1) represents the QFEB formula in the center-of-mass system of the projectile and the orbital electron. In order to compare with the experiment, this equation must be changed to that in the laboratory system by Lorentz transformation as expressed by\(^{13} \)

\[
\sigma_{QFEB}^{Lab}(T_r, h_\omega, \theta_L) = \frac{1 - \beta^2}{1 - \beta \cos \theta} \sigma_{QFEB}^{CM}(T_r, h_\omega, \theta), \quad (5)
\]

\[
h_\omega = \frac{1 - \beta \cos \theta}{\sqrt{1 - \beta^2}} h_\omega^L,
\]

and

\[
\cos \theta = \frac{\cos \theta_L - \beta}{1 - \beta \cos \theta_L},
\]

Equation (5) has been obtained by assuming that the orbital electron is free and at rest.

Now, we will derive the QFEB formula for a free electron having the velocity components \((V_x, V_y, V_z)\). The relative kinetic energy \( T_r' \) of the electron with respect to the projectile is

\[
T_r' = T_r \left( \frac{V_x}{p} \right)^2 + \left( \frac{V_y}{p} \right)^2 + \left( 1 - \frac{V_z}{p} \right)^2, \quad (6)
\]

where the \( z \)-axis is taken in the direction of incident beam. In the case of \( V_p > V_e \), only \( V_z \) of the velocity components is effective to \( T_r' \), and Eq. (6) can by approximated by

\[
T_r' \approx T_r \left( 1 - \frac{V_z}{p} \right)^2. \quad (7)
\]
The velocity-distribution function of orbital electrons in the $z$-direction can be obtained by

$$
\rho(V_z)dv_z = \int \rho(V_x, V_y, V_z)dv_xdv_y.
$$

(8)

Here, $\rho(V_x, V_y, V_z)$ is the velocity-distribution function of orbital electrons; it is given for the 1s electrons by

$$
\rho_{1s}(V_z)dv_z = \frac{8}{3\pi} \frac{V_0^5dv_z}{V_0^2V_z^2Z_1^3},
$$

where $\frac{1}{2}m_eV_0^2$ is the binding energy for the 1s state. From the discussion described above and from Eqs. (5), (7) and (8), the QFEB formula taking account of the velocity distribution of orbital electrons is expressed by

$$
\sigma_{QFEB}^{\text{QFEB}}(\omega_L', \theta_L') = \int_{-\infty}^{\infty} \sigma_{\text{QFEB}}^{\text{Lab}} (T_r', \omega_L', \theta_L') \rho(V_z)dv_z,
$$

(9)

Here, the upper limit of the integration with respect to $V_z$ is determined from the condition $T_r' \geq \omega_L$.

In Fig. 1, we can obviously distinguish two components of the continuum $x$ rays; the one is QFEB and the other one which extends to high-energy region is considered to be mainly SEB. Since it has been confirmed that $\omega_L^{\text{SEB}}(\theta_{L'}, \omega_{L'})$ is a monotonous function of $\omega_{L'}$ (5,7) SEB is approximately expressed by

$$
\omega_{L'} \omega_{L}^{\text{SEB}}(\theta_{L'}, \omega_{L'}) = \sum_{n=0}^{\infty} a_n(\theta_{L'})(\log \omega_{L'})^n.
$$

(10)

The coefficients $a_n$ can be determined from least-squared fitting to the experimental spectral region where QFEB does not contribute. Here, the order $n$ in Eq. (10) is taken as $n \leq 2$; this gives the most reasonable fit. The curves $\omega_{L}^{\text{SEB}}(\theta_{L'}, \omega_{L'})$ thus determined are shown in Fig. 1 as the background for QFEB. As the result of this estimation of the background, we are able to discuss quantitatively QFEB.

In Fig. 1, the curves which agree well with the experimental QFEB are the sum of SEB calculated from Eq. (10) and QFEB calculated from Eq. (5). It can be seen in this figure that the agreement between the theory and the experiment is quite satisfactory except the region near the end point of QFEB. In particular, the angular dependence of the QFEB spectrum originating from the Doppler effect $-\omega_{\text{end}} = T_r'1-\beta^2/(1-\beta\cos\theta_L')$ is well reproduced by the theory. Angular distributions of QFEB obtained by subtracting the SEB are shown in Fig. 2, together with those calculated from Eq. (5). Here again, the agreement between the experiment and the theory is quite good except the region near the end-point energy. The production cross section for QFEB given by Eq. (5) is discontinuous at the end-point energy $\omega_{\text{end}}$ because of the correction for
Coulomb deflection — \( g(\xi_0, \xi) \). As seen in Fig. 1, however, the experimental spectra of QFEB near the end-point energy continue to a region higher than \( T_{\text{end}} \) and gradually decrease to zero. In order to interpret this behavior, first the energy spread of incident beam is to be considered. Proton beams of 20 MeV bombarded the Be target of 46 mg/cm\(^2\), so that the effective energy and spread in the target are estimated to be 19.22 \pm 0.78 MeV at \( \theta_L = 90^\circ \). This energy spread of the proton beam results in the spectral spread of only \( \pm 0.42 \) keV at the end-point energy of QFEB. Next, the spectrum of QFEB obtained at \( \theta_L = 90^\circ \) is compared with theoretical calculations in Fig. 3, where the dot and dashed curve is calculated from Eq. (5) and the solid curve is obtained from Eq. (8) taking account of the velocity distribution of orbital electrons. It is found in this figure that Eq. (8) is in excellent agreement with the experiment and the velocity distribution of orbital electrons gives considerable effect on the spectral shape near the end-point. As in the case of radiative electron capture,\(^{15} \) QFEB is therefore a process which sensitively reflects the velocity distribution of orbital electrons and is expected to be useful to obtain information on orbital electrons.

References


Fig. 1. Continuum x-ray spectra for the Be target bombarded with 20-MeV protons obtained over the angular range $\theta_L = 50^\circ - 148^\circ$. The ordinate shows the cross section multiplied by the x-ray energy. The smooth curves in the region 3-30 keV are obtained from least-squared fitting of Eq. (10) to the experimental results. The other curves show the sum of cross sections for SEB and for QFEB calculated from Eq. (5).
Fig. 2. Angular distributions of QFEB. The solid curves are theoretical predictions obtained from Eq. (5).

Fig. 3. The QFEB spectrum near the end-point energy obtained at \( \theta_L = 90^\circ \) by subtracting the SEB calculated from Eq. (10). The dot and dashed curve shows the prediction from Eq. (5) and the solid curve was calculated from Eq. (8) by taking account of the velocity distribution of orbital electrons.