V. 14 On the Representation of Transmission Curves for Low-Energy Electrons through Thin Foils

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In low-energy conversion-electron or β-ray spectroscopy electrons are usually detected by gas counters having a thin window for an easy transmission of low-energy electrons. In this case we require the transmission curve of electrons as a function of electron energy for a given window thickness in order to correct for the detection efficiency of a counter. The energy region of electrons we are interested in is a few keV to several tens keV. For the transmission curves of this energy range we usually consult the extensive experimental data of Lane and Zaffarano, which are given in graphical form\(^1,2\)}, from which we must make interpolation for an appropriate window thickness. In order to avoid this somewhat cumbersome procedure we tried to represent the transmission curves in terms of analytical expressions.

Two-Parameter Representation

Recently we utilized a two-parameter representation of the transmission curve, \(T(E)\), in analysing the low-energy conversion spectrum in \(\text{\^{206}Bi}\,\text{I}\) in which the fraction of absorption, \(A = 1 - T\), is represented as

\[
A = 1 - T = a E_e^{-b},
\]

(1)

where \(E_e\) is the electron energy in keV. The parameters \(a\) and \(b\) are calculated from \(E_{90}\) and \(E_{50}\) as

\[
b = \ln 5/\ln (E_{90}/E_{50}) \quad \text{and} \quad a = 0.5 (E_{50})^b.
\]

The \(E_{90}\) and \(E_{50}\) are the electron energies at which \(T = 90\% \) and \(50\%\), respectively; these values are found to be calculated from the practical energy of electron cut-off, \(E_p\), corresponding to the extrapolated range of the electron through foils [see eq. (5) below].

Three-Parameter Representation

In the experimental transmission curve, \(T_{\text{exp}}\), there is a linear portion for \(0.1 \leq T_{\text{exp}} \leq 0.5\) and the extrapolation of this portion to \(T_{\text{exp}} = 0\) defines \(E_p\), and eq. (1) cannot represent such a situation. Moreover, for \(T_{\text{exp}} \leq 0.2\) (i.e., \(T_{\text{exp}} \geq 0.8\)) the experimental curve tends to deviate from a straight line as described by eq. (1) in a log-log plot. To consider these points a third parameter, \(c\), is introduced to represent \(A\). We put

\[
A = 1 - T = a (E_e - c)^{-b}
\]

(2)
and require that the curve be tangent to a straight line connecting the point of \((E_t', T(E_t'))\) and \((E_p', 0)\). Choosing \(E_t = E_{50}'\), we have

\[
\left(\frac{dT}{dE}\right)_{E_{50}'} = ab(E_{50}' - c)^{-b-1} = 0.5/(E_{50}' - E_c')
\]

(3)

together with

\[
a(E_{90}' - c)^{-b} = 0.1 \text{ and } a(E_{50}' - c)^{-b} = 0.5.
\]

(4)

The \(E_{90}'\) and \(E_{50}'\) can be represented as

\[
\begin{align*}
E_{90}' &= 0.413 \left(E_p' + 3.3\right)^{1.548} \text{ and} \\
E_{50}' &= 0.659 \left(E_p' + 1.6\right)^{1.240}.
\end{align*}
\]

(5)

Eqs. (3) and (4), in which eq. (5) is used, were solved numerically for the practical energy \(E_p\) of the foil samples of ref. 1 and also for some selected values of \(E_p\). The results are given in Fig. 1 (for \(E_p\) of ref. 1) and in Table 1 (for representative \(E_p\)'s). The represented transmission curves corresponding to Table 1 are given in Fig. 2. Fig. 1 shows that the present three-parameter representation reproduces the experimental transmission curves within an error of \(\pm 2\%\) for \(T \geq 50\%\) for a wide range of window thickness; No. 1 and No. 11 samples correspond to window thicknesses of \(1.57\ \text{mg/cm}^2\) and \(24\ \mu\text{g/cm}^2\), respectively. For \(T \geq 90\%\) the fitting error is smaller than \(\pm 1\%\). In Table 1 the extrapolated range \(R_{ex}\) corresponding to \(E_p\) estimated by the Tabata's formulae\(^4\) for foils of polypropylene \((\text{C}_3\text{H}_6)_n\), which is suitable for counter windows, is also given. The numerical solution of eqs. (3) and (4) was made by an iterative method, which turned out to have a slow convergence, especially, for smaller \(E_p\); this is reflected in large absolute values of the three parameters in Table 1.

We also tried a Fermi-type function,

\[
T = a + \frac{1-a}{1+\exp[-b(E_c'-c)]},
\]

together with a tangential condition similar to eq. (3), but we found that the experimental transmission curves cannot be fitted in this way except for \(E_p \leq 1.2\ \text{keV}\) [eq. (5) was also used].

References

3) Fujisoka M. et al., "Internal Conversion of Valence-Shell Electrons: Measurement and Analysis for the 10.84 keV Transition in \(^{206}\text{Bi}\)", to be published.
Table 1. Parameters of three-parameter representation of transmission curves of electrons through thin foils

<table>
<thead>
<tr>
<th>E_p (keV)</th>
<th>E_{50} (keV)</th>
<th>E_{90} (keV)</th>
<th>a</th>
<th>b</th>
<th>c(keV)</th>
<th>Extrapolated range</th>
<th>range^a</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>2.468</td>
<td>4.384</td>
<td>5.244(67)</td>
<td>40.59</td>
<td>-44.92</td>
<td>11.5</td>
<td>0.127</td>
</tr>
<tr>
<td>2.0</td>
<td>3.226</td>
<td>5.459</td>
<td>5.738(5)</td>
<td>6.649</td>
<td>-4.927</td>
<td>19.5</td>
<td>0.216</td>
</tr>
<tr>
<td>3.0</td>
<td>4.372</td>
<td>7.134</td>
<td>2.206(2)</td>
<td>3.730</td>
<td>-0.746</td>
<td>33.3</td>
<td>0.362</td>
</tr>
<tr>
<td>4.0</td>
<td>5.580</td>
<td>8.961</td>
<td>4.718(1)</td>
<td>2.952</td>
<td>0.915</td>
<td>49.9</td>
<td>0.551</td>
</tr>
<tr>
<td>6.0</td>
<td>8.149</td>
<td>13.04</td>
<td>2.951(1)</td>
<td>2.453</td>
<td>2.877</td>
<td>91.2</td>
<td>1.01</td>
</tr>
<tr>
<td>10.0</td>
<td>13.77</td>
<td>22.68</td>
<td>5.432(1)</td>
<td>2.212</td>
<td>5.437</td>
<td>206</td>
<td>2.28</td>
</tr>
<tr>
<td>15.0</td>
<td>21.47</td>
<td>37.17</td>
<td>1.130(2)</td>
<td>2.084</td>
<td>7.989</td>
<td>410</td>
<td>4.53</td>
</tr>
<tr>
<td>20.0</td>
<td>29.76</td>
<td>54.03</td>
<td>1.718(2)</td>
<td>1.974</td>
<td>10.50</td>
<td>679</td>
<td>7.51</td>
</tr>
<tr>
<td>50.0</td>
<td>87.61</td>
<td>194.5</td>
<td>2.564(2)</td>
<td>1.538</td>
<td>29.77</td>
<td>3550</td>
<td>39.2</td>
</tr>
</tbody>
</table>

^a) For polypropylene (C_3H_6)_n with \( \rho = 0.905 \text{ g/cm}^3 \) calculated according to ref. 4.
1) Comparison of the experimental transmission curves of electrons through thin foils with those represented by the three-parameter function; see eqs. (3), (4) and (5). The sample number and the corresponding practical energy of electron cut-off, $E_p$, are indicated.

2) Transmission curves calculated using the three-parameter representation for the parameters of Table 1. The practical energy of electron cut-off, $E_p$, is indicated. For $T \leq 50\%$ the curves are straight lines in linear plot intersecting the abscissa at $E_p = E_p$, and the dotted curves are extrapolations of the three-parameter function to $T \leq 50\%$; note that the two curves have a contact at $T = 50\%$. 