I. 9. Preliminary Results on Estimating the Attenuation Coefficients in PET using the Single Events

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Introduction

During a Positron Emission Tomography (PET) scan, there are several physical effects arising from the interaction photon-matter. The main two interactions are attenuation and scattering of photons. Their effects on total coincidence counting should be evaluated and corrected, if reliable quantitative results are a goal\(^1\). The photon attenuation has several detrimental effects like overall loss of counts, higher image noise, image non-uniformity due to non-uniform photon attenuation. Overall losses of counts in the body due to attenuation can be as high as 85% for a moderately large body, but the loss varies substantially with the body size, resulting in varying noise levels\(^2\). Development of attenuation correction techniques is a very active field in PET and, recently, it has been developed a scanner that allows simultaneous acquisition of attenuation images (X-ray CT) and PET. In general, the choices for attenuation correction can be divided in two groups: calculated and measured attenuation correction\(^3\). The first one uses an image segmentation technique and assigns a uniform attenuation coefficient to each segment\(^4\). The measured attenuation is performed by scanning the patient with an external radiation source (transmission scan), thus the total attenuation for each coincidence line (LOR) is measured. These methods have advantages and disadvantages. The calculated attenuation is easy to apply and does not require a transmission scan. However, the assumption that the segments have homogeneous attenuation properties, leads to errors in some types of scan. For brain imaging it gives good results, but the skull attenuation properties cannot be completely recovered\(^5\). The transmission scan detect attenuation discontinuities, which is desirable for whole body or thorax imaging. However, it introduces statistical errors since transmitted
photons must also be measured. This can be improved by high counting but it will imply either larger scanning time or higher activity transmission sources. This research presents the preliminary results on determining the attenuation properties of the imaged object by using the single event rates. Only simulated data were used for this report.

**Description of the method**

The conventional PET images are reconstructed from coincidence events originated from electron-positron annihilation. However, if one of the photon forming the coincidence pair is absorbed or scattered out of the gantry, only one photon will be counted, hence producing a single count. Several PET systems can measure the single events for each detector, since they are useful for estimating the accidental coincidence. Moreover, the single events also carry valuable information on the attenuation properties of the imaged object.

It is assumed to have a ring system with M (m=1,…,M) detectors defining L (l=1,…,L) coincidence pairs, as shown in Fig.1. The image is divided into B (b=1,…,B) image elements. Each image element is thought of as having an average attenuation coefficient \( \mu_b \). Each radiation detector \( m \) counts \( S_{data}^{\text{data}} \) single events and each coincidence pair \( l \) receives \( \rho_l \) coincidence counts, however due to the effect of the dead time (\( \tau \)) it counts \( \gamma_l \) events. Assuming a paralyzable behavior of the dead time, it is known that:

\[
\gamma_l = \rho_l \exp(-\rho_l \tau)
\]

(1)

It is defined \( p_{mb} \) and \( c_{lb} \) as the geometrical probability for single and coincidence detection, respectively. \( g_{bb'} \) are the elements of the inverted \( L \times B \) matrix defined by \( c_{lb} \). A simple work on the mathematics involved in the coincidence and single detection models, it is reasonable to say that the mathematical model that allows us to determine the attenuation properties of the imaged object is written as:

\[
S_m^{\text{model}} = \sum_{b=1}^{B} \sum_{b'=1}^{B} \sum_{l=1}^{L} p_{mb} A_{mb} g_{bb'} c_{lb'} \rho_l A_l^{-1}
\]

(2)

where \( A_{mb} = \exp\left(-\sum_{b'=1}^{B} \mu_{b'} r_{mb'} \right) \) and \( A_l = \exp\left(-\sum_{b'=1}^{B} \mu_{b'} r_{lb'} \right) \). It means that the single counting can be modeled by the coincidence events (after dead time correction), attenuation properties of the object and the geometrical characteristics of the imaging system. The
determination of attenuation properties can be treated as a least square minimization problem where the figure to be minimized is

$$\sum_{m=1}^{M} \left[ \frac{S_{m}^{data} - S_{m}^{model}}{\sigma_{m}^{data}} \right]^{2}$$  \hspace{1cm} (3)

The Newton-Raphson method for non-linear systems of equations was tested in a first trial to solve the non-linear system provided by (3). This method needs a function $F_{m}$ is defined as:

$$F_{m} = \frac{S_{m}^{data} - S_{m}^{model}}{\sigma_{m}^{data}}$$  \hspace{1cm} (4)

The objective is to minimize $F_{m}$. It can be achieved setting $F_{m}=0$. The Taylor expansion series tell us that

$$F_{m}(\mu + \Delta \mu) = F_{m}(\mu) + \sum_{b=1}^{k} \frac{\partial F_{m}}{\partial \mu_{b}} \Delta \mu_{b}$$  \hspace{1cm} (5)

where

$$\mu_{b} = \begin{bmatrix} \mu_{1} \\ \mu_{2} \\ \vdots \\ \mu_{B} \end{bmatrix}$$  \hspace{1cm} (6)

The iterative algorithm to update each $\mu_{b}$ until $F_{m}$ reaches a minimum value is written as:

$$\Delta \mu = - \frac{F}{J}$$  \hspace{1cm} (7)

where $F$ is the vector of functions $F_{m}$ and $J$ is the jacobian of $F$ respect to $\mu$. Therefore,

$$\mu^{new} = \mu^{old} + \Delta \mu$$  \hspace{1cm} (8)

When the iterative process converges, the resulting vector $\mu$ will represent a rough estimation of the attenuation properties of the imaged object.

**Simulations**

In order to test the convergence of the described method, two different distributions of attenuation coefficients (cm$^{-1}$) were used to simulate the coincidence and
single data. These data were used as input for the iterative process which estimates the attenuation coefficients that were previously used to simulate the data. The image was divided into a 4 x 4 matrix. It means the image has of 16 image elements (pixels). An attenuation coefficient and activity value is assigned to each pixel. The distributions are sketched in Fig. 2.

The distribution A is uniform (the attenuation coefficients are the same for every image pixel). Several coefficients were tested for this distribution. The distribution B shows different attenuation coefficients for different pixels. Several values were tested for this distribution as well. The activity distribution chosen for data simulation was to assign an activity of 750000 cps to the four central pixels. The activity for all other pixels is assumed to be zero.

**Results and Discussion**

The results are summarized in the table 1. The main interest is to know if the iteration process converged to the expected attenuation coefficients.

It is clear that this method does not converge for high attenuation coefficients (>0.25 cm\(^{-1}\)). This tendency does not depend on the distribution. However, this technique is mainly addressed for brain imaging where the attenuation coefficients are not so large, except for the skull (around 0.26 cm\(^{-1}\)). Therefore, the probability to have a high attenuation coefficient is low since the attenuation coefficient for each pixel represents the average attenuation within the pixel whose size is very large. In general, the iterative algorithm converged after a maximum of 12 iterations and the results showed a maximum deviation of 5% with respect to the expected values.

**Conclusion**

This report describes the preliminary results for a method aiming to estimate the attenuation correction coefficients for each LOR by using the single photon counting rate for each detector. The method is an iterative algorithm based on the Newton-Raphson method for solving non linear equations. For cases where simulated data was obtained using attenuation coefficients larger than 0.25 cm\(^{-1}\), the method did not converge to the expected attenuation coefficients. At this point we do not have an explanation for this behavior. This method clearly offers a very rough estimation of the attenuation coefficients compared to the transmission scan. However, attenuation coefficients within the brain do not change too drastically. Hence, this technique could give satisfactory estimations for
brain imaging. As presented in this report, the technique has several restrictions. The first one is the method is only useful for 2D imaging. Extension to 3D imaging is only possible if the Compton scattering effect is included. For 2D imaging, scatter photons contribution is around 10-15% which is expected to cause a small deviation from the true attenuation coefficients. However, scatter contribution increases to 40% in 3D imaging. If this effect is not modeled, the estimated values will not be reliable. The second restriction is related to the scanner hardware. This technique is only useful if the scanner provides simultaneously, single and coincidence rates.

References

2) Harkness B., Comparison of currently available PET scanners, Society of Nuclear Medicine, 47th annual meeting, June 2000.

Table 1. Results of the iteration for estimating the attenuation coefficients.

<table>
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Fig. 1. 2D-PET scanner and discretized imaged.

Fig. 2. Distribution of attenuation coefficients in the imaged object.